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Minimum Work as a Possible Criterion for Determining the Frictional Conditions at the Tool/Chip Interface in Machining

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MINIMUM WORK AS A POSSIBLE CRITERION FOR DETERMINING THE FRICTIONAL CONDITIONS AT THE TOOL/CHIP INTERFACE IN MACHINING

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[Plate 1]

CONTENTS

An approximate theory of machining is described in which the average shear flow stress in the plastic zone in the chip adjacent to the tool/chip interface, which is allowed to vary with strain rate and temperature, is used as the friction parameter and this is shown to be far more effective than the normally used average coefficient (or angle) of friction. It is proposed that the average thickness of the tool/chip interface plastic zone is determined by a minimum work criterion, its value being such that for given cutting conditions the average shear flow stress within the plastic zone will be minimized, thus minimizing both the frictional and total work done in chip formation. A comparison is made between results predicted by assuming minimum work and experimental results.

1. Introduction

In analysing the mechanics of metal machining it has been usual – for example, Merchant (1945), Lee & Shaffer (1951) – to define the frictional conditions at the tool/chip interface in terms of an average coefficient of friction μ or angle of friction λ with

$$\tan \lambda = \mu = F/N, \tag{1}$$

where F (figure 1) and N are the friction and normal forces at the tool/chip interface. Values of λ calculated from the equation (derived from the force geometry in figure 1)

$$\tan (\lambda - \alpha) = F_T/F_C, \tag{2}$$

where α (figure 1) is the tool rake angle, and F_C and F_T are the forces in the cutting direction and normal to this direction, using experimentally measured cutting forces have shown that λ varies markedly even for a given work material/tool material combination with such cutting conditions as cutting speed, tool rake angle and undeformed chip thickness (t_1 in figure 1). Although the variations with cutting speed can be partly explained in terms of the normal theories of sliding friction, these theories cannot explain the variations with α and t_1. In considering this problem a number of workers, including Zorev (1963) and Wallace & Boothroyd (1964), have pointed out that over much of the contact length between chip and tool the real and apparent areas of contact are equal, with plastic deformation occurring in the bulk material in the chip adjacent to the tool/chip interface. If this is compared with the conditions encountered in normal sliding friction (and assumed in the corresponding theories) where, for metals, the surfaces contact only at local asperities and the real area of contact is only a small fraction of the apparent area of contact, it is not surprising that λ in machining does not follow the usual laws of sliding friction.

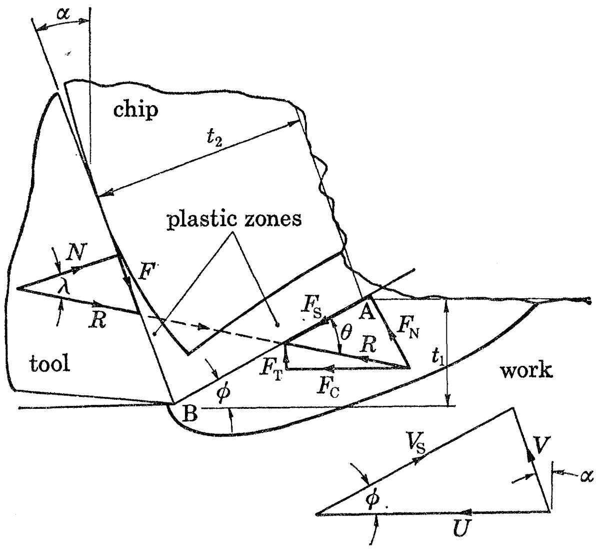

FIGURE 1. Model of chip formation used in analysis.

This paper starts by describing an approximate theory of machining in which λ is replaced as the friction parameter by the average shear flow stress within the plastic zone adjacent to the tool/chip interface which can vary with strain rate and temperature. Then using flow stress results obtained from high speed compression tests, made over a wide range of temperatures, to represent the work material's flow stress characteristics the machining theory is applied to predict cutting forces, etc., and a comparison is made between predicted values of λ obtained in this way and experimental values obtained from equation (2) using measured cutting forces. In the machining theory it has been found necessary to use a number of empirical expressions including one for the average thickness of the tool/chip interface plastic zone which must be known in order to determine the associated strain rates and temperatures, and the main purpose of the present paper is to show that this thickness is apparently determined by a minimum work criterion. A comparison is made between tool/chip interface plastic zone thicknesses and chip thicknesses predicted by assuming minimum work and experimental values measured from chip sections.

2. Machining theory

The analysis is for orthogonal machining in which a surface layer of material is removed by a tool with a single, straight cutting edge which is set normal to the cutting velocity. If the undeformed chip thickness t_1 (figure 1) is small compared with the width of cut measured along the cutting edge, then the removed chip is formed under approximately plane strain conditions and attention is limited to such conditions. A further restriction is that the chip is assumed to be formed by plastic deformation with no cracking and with no build-up of work material on the cutting tool, that is, under steady state conditions.

The machining theory to be described has been developed over a number of years. The early work of Christopherson, Oxley & Palmer (1958) and Palmer & Oxley (1959) was based on very slow speed machining tests in which cine films were taken through a microscope of the plastic flow occurring in chip formation. A slip-line field analysis of this observed flow showed that in order to satisfy stress and force equilibrium, account had to be taken in the stress equilibrium equations of variations in flow stress, in this case with strain. More recent work, including that of Fenton & Oxley (1969) and Hastings, Oxley & Stevenson (1974), which has led to the development of a predictive theory of machining, has confirmed the importance of allowing for variations in flow stress and has shown that at the high cutting speeds used in practice, account must be taken of the influence of strain rate and temperature as well as strain on flow stress.

The model of chip formation used in the analysis is given in figure 1. In this the plane AB is found from the same construction as used in the well-known Merchant (1945) shear plane model of chip formation. With the shear plane model the rigid work velocity U is changed to the rigid chip velocity V instantaneously at AB, which requires a discontinuity in tangential velocity across AB and which can be represented by the velocity diagram given in figure 1 with the velocity discontinuity equal to V_S. The shear plane defined in this way can be assumed to be a direction of maximum shear strain rate (infinite in this case) and hence from isotropic plasticity theory also a direction of maximum shear stress. With the present model (figure 1) AB can again be assumed to be a direction of maximum shear strain rate and maximum shear stress, but in this case it must be considered as a slip-line near the centre of a finite plastic zone in which the change from velocity U to velocity V occurs along smooth streamlines. Experiments (Stevenson & Oxley 1970) have shown, however, that the same velocity diagram (figure 1) as used with the shear plane model still represents the overall change in velocity in this zone with sufficient accuracy. The tool/chip interface is also assumed to be a direction of maximum shear strain rate and maximum shear stress† with plastic flow occurring in the bulk chip material over the entire contact length between chip and tool. The relatively small elastic contact length which occurs in the region just before the chip curls away from the tool is therefore neglected. These assumptions regarding the interface would be expected to be approached most closely at practical cutting speeds where the temperatures at the interface are relatively high and it is with such conditions that the present paper is most concerned. In essence the method of analysis is to determine the angle ϕ (figure 1) made by AB with the cutting direction, the so-called shear angle, and hence the chip thickness and cutting forces, etc., by analysing the stresses along AB

† This does not mean that AB must necessarily be normal to the tool/chip interface as there can be large changes in the curvature of the slip-lines in the region of B as shown by Roth & Oxley (1972) with correspondingly large changes in stress, thus allowing the boundary conditions in this region to be satisfied.

and the tool/chip interface and by selecting ϕ so that the corresponding resultant forces are in equilibrium.

Using printed grids to measure the plastic flow in chip formation Stevenson & Oxley (1970) have shown that the strain rate distribution is very nearly symmetrical about AB with the maximum value occurring at AB and that for given conditions the shear strain rate along AB varies little for most of its length, although it tends to increase in the region of B. Their results show that for a wide range of strain rate (1–10^5/s) the average strain rate along AB can be estimated from the empirical equation

$$\dot{\gamma}_{\mathrm{AB}} = C(V_{\mathrm{S}}/l), \tag{3}$$

where C is a material constant, V_{S} (figure 1) is the shear velocity and $l = t_1/\sin\phi$ is the length of AB. The results also show that because of the symmetry of the strain rate distribution about AB, approximately half the total strain has occurred at AB and that the shear strain along AB is given by

$$\gamma_{\mathrm{AB}} = \frac{1}{2}\frac{\cos\alpha}{\sin\phi\cos(\phi-\alpha)}. \tag{4}$$

The shear stress along AB is the maximum shear stress (shear flow stress) k, and if it is assumed that the temperature along AB (as well as the strain rate and strain) is constant for given conditions then $k = k_{\mathrm{AB}}$ can be assumed constant along AB. The stress normal to AB is the hydrostatic stress p and its distribution along AB can be determined by starting at the free surface in the region of A (assumed to be parallel to the cutting velocity) and by applying the appropriate stress equilibrium equation (i.e. $\mathrm{d}p = (\mathrm{d}k/\mathrm{d}s_1)\,\mathrm{d}s_2$, where s_1 and s_2 are distances measured normal to and along AB). Combining these stress distributions it can be shown that

$$\tan\theta = 1+2(\tfrac{1}{4}\pi-\phi)-Cn, \tag{5}$$

where θ (figure 1) is the inclination of the resultant cutting force R to AB, C is the constant in equation (3) and n is the strain-hardening index in the empirical stress–strain relation

$$\sigma = \sigma_1\epsilon^n, \tag{6}$$

where σ and ϵ are the uniaxial flow stress and strain and σ_1 and n are material 'constants' which define the stress–strain curve for given values of strain rate and temperature. It should be noted that in deriving equation (5) it is assumed in applying the stress equilibrium equation that the flow stress gradient normal to AB results only from the corresponding strain gradient (i.e. $\mathrm{d}k/\mathrm{d}s_1 = (\mathrm{d}k/\mathrm{d}\gamma)\,(\mathrm{d}\gamma/\mathrm{d}s_1)$) and no account is taken of the strain rate gradient (which is approximately zero as strain rate passes through a maximum near to AB) or temperature gradient. The reason for neglecting the temperature gradient is that it is not easy to estimate its value, although recently Tay, Stevenson & de Vahl Davis (1974) have done this by analysing experimental flow fields using numerical techniques. Some compensation for neglecting the temperature gradient is obtained from the use of experimental flow stress data in equation (6) which are derived, for example, from high-speed compression tests in which there will be a temperature gradient of at least the same sign as that across AB. The results of Tay *et al.* suggest that neglect of the temperature gradient should not lead to large errors in the predicted machining results.

From the geometry of figure 1 the angle θ can also be expressed in terms of other angles by the equation

$$\theta = \phi+\lambda-\alpha, \tag{7}$$

where ϕ is the shear angle, λ is the average angle of friction defined as in equation (1) and α is the tool rake angle. Assuming that the tool is perfectly sharp so that the resultant force transmitted by AB is also transmitted by the tool/chip interface, the following geometric force relations can be obtained

$$\left.\begin{aligned} F_C &= R\cos(\lambda-\alpha),\\ F_T &= R\sin(\lambda-\alpha),\\ F &= R\sin\lambda,\\ R &= \frac{F_S}{\cos\theta} = \frac{k_{AB}\,t_1\,w}{\sin\phi\cos\theta},\end{aligned}\right\} \tag{8}$$

where F_S is the shear force along AB and w is the width of cut measured along the tool cutting edge.

The mean temperature rise in the plastic zone in which the chip is formed is found by considering the plastic work done in this zone and is given by

$$T_{SZ} = \frac{1-\beta}{\rho S t_1 w}\frac{F_S\cos\alpha}{\cos(\phi-\alpha)}, \tag{9}$$

where ρ is the density of the work material, S its specific heat and β is the proportion of heat conducted into the work. There have been a number of attempts to predict β theoretically including that by Weiner (1955) but the authors have found that the most reliable estimates of β and hence temperatures can be made using the following empirical equations based on a compilation of experimental data made by Boothroyd (1963)

$$\left.\begin{aligned} \beta &= 0.5-0.35\lg(R_T\tan\phi) \quad \text{for} \quad 0.04 \leqslant R_T\tan\phi \leqslant 10.0,\\ \text{and}\quad \beta &= 0.3-0.15\lg(R_T\tan\phi) \quad \text{for} \quad R_T\tan\phi > 10.0,\end{aligned}\right\} \tag{10}$$

with R_T a non-dimensional thermal number given by

$$R_T = \rho S U t_1/K, \tag{11}$$

where U (figure 1) is the cutting speed and K the thermal conductivity of the work material. The further limits that β should not exceed 1 or be less than 0 are also imposed. The average temperature along AB is taken as

$$T_{AB} = T_{SZ}+T_W, \tag{12}$$

where T_W is the initial work temperature.

Considering the tool/chip interface the average strain rate in the plastic zone adjacent to the interface is assumed to be given by

$$\dot{\gamma}_{int} = V/\delta t_2, \tag{13}$$

where V (figure 1) is the chip velocity and δ is the ratio of the average thickness of the plastic zone to the chip thickness t_2. This equation implies that the velocity changes from zero at the tool cutting face to V over the plastic zone thickness δt_2 when for most cutting conditions there will be some sliding by the chip over the cutting face.† This will lead to equation (13) over-estimating the strain rate but the error should in most cases be small. Originally Fenton &

† In fact for steady-state conditions material at the cutting face must leave the plastic zone with a velocity equal to the rigid chip velocity. As shown by Roth & Oxley (1972) this means that the sliding velocity increases along the cutting face, which cannot therefore be a direction of maximum shear strain rate as there is a direct strain rate in this direction. This will, however, be very small compared with $\dot{\gamma}_{int}$ and the cutting face will approximate closely to a direction of maximum shear strain rate as assumed in the analysis.

Oxley (1969) derived an equation for the length of tool–chip contact by calculating the position of the resultant force from the stresses on AB and then assuming that the resultant force would cut the tool cutting face a distance from the cutting edge equal to a third the total contact length. This is equivalent to assuming a triangular distribution of normal stress along the tool cutting face with a maximum at the cutting edge. More recently Hastings *et al.* (1974) have found that better estimates of the contact length can be made from the equation

$$h = \frac{t_1 \sin \theta}{\cos \lambda \sin \phi}, \tag{14}$$

which is determined by drawing a line parallel to the resultant cutting force through A (figure 1) and letting the point where this line cuts the tool cutting face determine h. This equation has been proposed a number of times before and in justifying its use it has been argued that any force at the tool/chip interface beyond this point will not be transmitted directly to the rigid work and will therefore lack support. It might therefore be reasonably expected that stress levels will diminish rapidly over any contact greater than given by equation (14) which will, however, tend to underestimate the total contact length.

The average temperature at the tool/chip interface is assumed to be given by

$$T_{\text{int}} = T_{\text{AB}} + T_{\text{M}}, \tag{15}$$

where T_{M} is the maximum temperature rise in the chip. Using numerical methods Boothroyd (1963) has calculated T_{M} assuming a rectangular plastic zone (heat source) at the tool/chip interface and has shown that his calculated values are in good agreement with experimentally measured temperatures. Boothroyd's results can be represented by the empirical equation

$$\lg\left(\frac{T_{\text{M}}}{T_{\text{C}}}\right) = 0.06 - 0.195\,\delta \sqrt{\left(\frac{R_{\text{T}} t_2}{h}\right)} + 0.5 \lg\left(\frac{R_{\text{T}} t_2}{h}\right), \tag{16}$$

where T_{C}, which is the average temperature rise in the chip, is given by

$$T_{\text{C}} = \frac{F \sin \phi}{\rho S t_1 w \cos(\phi - \alpha)} \tag{17}$$

with all other terms as defined before.

The above equations are now sufficient to calculate cutting forces, temperatures, etc., for given cutting conditions so long as the appropriate work material properties and the values of C in equations (3) and (5) and δ in equations (13) and (16) are known.

3. Work material properties

There are very few flow stress data available for the extreme conditions of strain, strain rate and temperature encountered in machining (see, for example, Proceedings of Conference on Mechanical Properties at High Rates of Strain, Oxford (1974)) and because of this the first attempts to apply the machining theory to predict cutting forces, etc., were made using flow stress data obtained from machining results. The approach, which has been described by Fenton & Oxley (1970), is to take a small number of experimental machining results for cutting forces and shear angle and to use the machining theory in reverse to calculate σ_1 and n in equation (6) and the corresponding strain rates and temperatures. These flow stress results are then used with the machining theory to make predictions over a much wider range of cutting conditions. This rather unsatisfactory procedure has recently been overcome by Hastings *et al.* (1974) by using flow stress data obtained by Oyane *et al.* (1967) from high-speed compression tests for a range of plain carbon steels. In these the strain rate ($\approx$ 450/s) was lower than that usually encountered in machining (10^3–10^6/s) but the testing temperatures covered a wide range (0–1100 °C), which makes it possible to extrapolate the compression test results into the machining range by using the velocity modified temperature concept of MacGregor & Fisher (1946). This can be expressed in the form

$$T_{\text{mod}} = T(1-\nu \lg \dot{\epsilon}/\dot{\epsilon}_0), \tag{18}$$

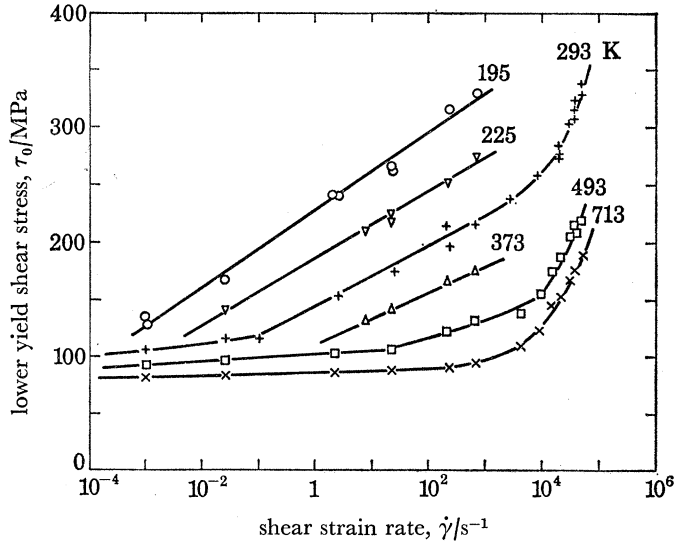

FIGURE 2. Experimental results (Campbell & Ferguson 1970) showing effects of strain rate and temperature on lower yield shear stress of a 0.12% plain carbon steel.

where T (K) is the temperature and $\dot{\epsilon}$ the strain rate with ν and $\dot{\epsilon}_0$ constants. It is assumed that for a given strain the flow stress for a particular material will be a unique function of T_{mod} defined in this way. This parameter has been used many times before to represent the opposing effects of strain rate and temperature but normally in situations where temperature is the main variable and there are relatively small variations in strain rate with the strain rates not very high. In machining, strain rates are very high and both temperature and strain rate vary over large ranges. The compression test results of Oyane *et al.* (1967) cannot be used to test the validity of equation (18) for such conditions because the strain rate in these tests was constant. However, some recent results of Campbell & Ferguson (1970) which show the effects of very wide ranges of strain rate and temperature on the lower yield point of a 0.12 % plain carbon steel and which are reproduced in figure 2 are suitable for this purpose. Lower yield stress points taken from the curves in figure 2 at strain rates 1, 10, 10^2, 10^3, 10^4 and 3.2×10^4/s are replotted against T_{mod} with (ν = 0.09 and $\dot{\epsilon}_0$ = 1/s) in figure 3 and can be seen to fit a single curve exceptionally well, thus giving some support to the use of equation (18) for conditions where strain rate and temperature vary over large ranges and are of the same order of magnitude as in machining.

Oyane *et al.* (1967) have given results for plain carbon steels with carbon contents in the range 0.16–0.55 % and their results for a 0.16 % carbon steel are given in figure 4.† In the strain range 0.2–0.4 straight lines of best fit have been drawn to approximate the curves for each temperature.

† These were the only results considered in the paper by Hastings *et al.* (1974).

That these lines are a good approximation (over a restricted strain range) supports to some extent the use of the linear logarithmic stress against strain relation of equation (6) in the machining analysis. A good fit is not obtained over the much wider strain range of the compression tests, chiefly because of the near adiabatic conditions in the tests which cause variations in flow stress with the resulting temperature rise in addition to those caused by strain hardening. In deriving the constants σ_1 and n for each test it was therefore decided to use the straight lines drawn for the 0.2 to 0.4 strain interval. The value of n was found directly from the slope and

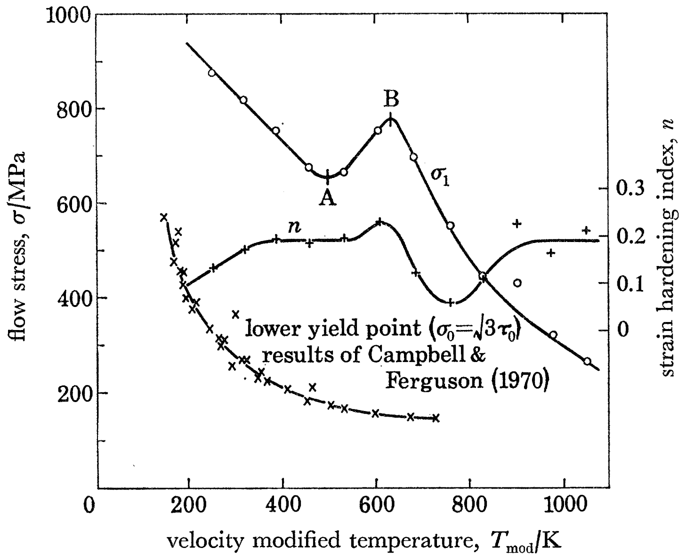

FIGURE 3. Flow stress results plotted against velocity modified temperature.

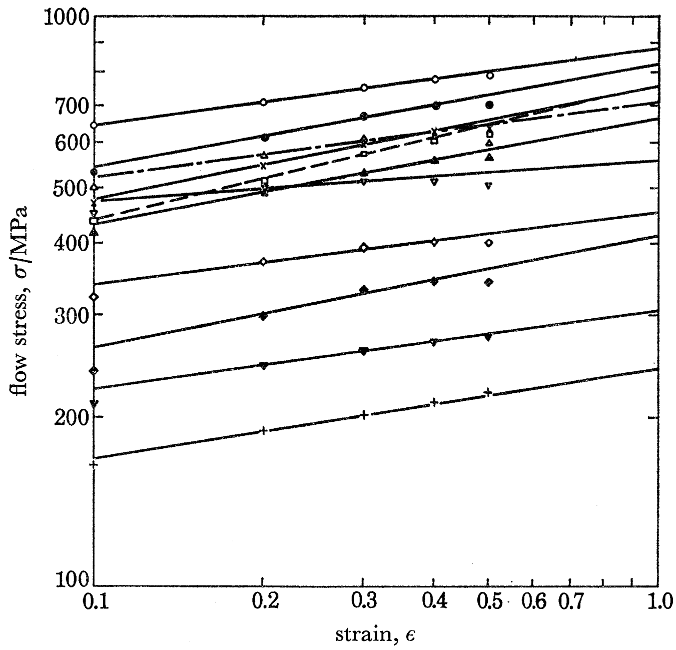

FIGURE 4. High-speed compression test results (Oyane *et al.* 1967) for a 0.16% plain carbon steel. ○, 0 °C; ●, 100 °C; ×, 200 °C; ▲, 400 °C; □, 500 °C; △, 600 °C; ▽, 700 °C; ◇, 800 °C; ◆, 900 °C; ▼, 1000 °C; +, 1100 °C.

σ_1 by extrapolating the line to a strain $\epsilon = 1$. In calculating the corresponding T_{mod} values the constants ν and $\dot{\epsilon}_0$ were taken as 0.09 and 1/s as before. The strain rate $\dot{\epsilon}$ was constant at 450/s and the temperature T (K) was taken as the test starting temperature plus a calculated temperature rise appropriate to the mid-point of the 0.2–0.4 strain interval. The temperature rise was calculated by assuming that all of the plastic work was converted to heat and that the conditions were adiabatic. The rise ΔT (K) was then given by

$$\Delta T = \frac{1}{\rho S}\int_0^{0.3} \sigma \, \mathrm{d}\epsilon, \tag{19}$$

where ρ is the density taken as 7862 kg/m^3 and S the specific heat calculated from the equation

$$S(\mathrm{J\,kg^{-1}\,K^{-1}}) = 420 + 0.504T, \tag{20}$$

where T (°C) is taken as the starting temperature.

The results for σ_1 and n for the 0.16 % carbon steel are plotted against T_{mod} in figure 3 and can be seen to show a clear dynamic strain-ageing (blue-brittle) region, where flow stress increases with increase in temperature, which is typical for this kind of steel. This effect is not evident in the results of Campbell & Ferguson (1970), which are also given in figure 3, because of the very low strain ($\epsilon < 0.01$) associated with these results. The curves for σ_1 and n have been represented mathematically using sections of polynomials and these have been used with rescaling functions to represent the σ_1 and n curves for the other carbon steels. The rescaling functions, which give an increase in σ_1 and a decrease in n with increase in carbon content, were chosen to give a good fit with the experimental values of σ_1 and n for the 0.35, 0.45 and 0.55 % carbon steels derived from the compression test results in the same way as described above. In this way continuous changes in σ_1 and n over the ranges of velocity modified temperature and carbon content considered are represented by a relatively simple set of functions.

In relating these uniaxial flow stress results to the plane strain machining conditions an effective stress, strain and strain rate (based on the shear strain energy yield criterion) are used in the usual way with the effective stress, for a given material, assumed to be a unique function of the effective strain, effective strain rate and temperature. Thus, for example, the following relations can be obtained.

$$\left.\begin{aligned} k_{AB} &= (\sigma_1/\sqrt{3})\, \epsilon_{AB}^{n}, \\ \epsilon_{AB} &= \gamma_{AB}/\sqrt{3}, \\ \dot{\epsilon}_{AB} &= \dot{\gamma}_{AB}/\sqrt{3}. \end{aligned}\right\} \tag{21}$$

The influence of carbon content on density and specific heat has been found to be small over the range considered and can be neglected. However, there is a marked influence on thermal conductivity K and allowance has been made for variations in K with carbon content and other alloying elements on the basis of experimental results (Woolman & Mottram 1964). The equations obtained in this way are similar to

$$K\ (\mathrm{W\,m^{-1}\,K^{-1}}) = 57.9 - 0.0345T \tag{22}$$

(where T is in °C), which is the equation used by Hastings *et al.* (1974) for the 0.16 % carbon steel.

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4. Comparison of predicted and experimental machining results

To predict the shear angle ϕ (figure 1) and the cutting forces, etc., for a given set of cutting conditions, the method used is to calculate for a range of values of ϕ the resolved shear stress at the tool/chip interface from the resultant cutting force obtained from the stresses on AB, and then for the same range to calculate the temperatures and strain rates at the tool/chip interface and hence the corresponding values of shear flow stress. The solution is taken as the value of ϕ which gives a shear flow stress in the chip material at the tool/chip interface equal to the resolved shear stress, as the assumed model of chip formation is then in equilibrium.

The given information will be the tool rake angle α, the cutting speed U, and the thickness t_1 and the width w of the undeformed chip, together with the thermal and flow stress properties of the work material and the initial temperature of the work. The values of C in equations (3) and (5) and δ in equations (13) and (16) must also be known. From experimental machining investigations on similar plain carbon steels to those now being considered Stevenson & Oxley (1970) and Stevenson & Duncan (1973) have found that C and δ remain approximately constant, with $C \approx 5.9$ and $\delta \approx 0.05$, and these are the values used in the calculations now described. It should be noted that while the experiments used in determining C covered a fairly wide range of cutting conditions this was not the case for δ where cutting speed was the only variable.

For a given ϕ equation (3) is used to calculate $\dot{\gamma}_{AB}$ (V_S and l are found from the geometry of figure 1) and equation (4) γ_{AB}. To find σ_1 and n from the equations used to represent the flow stress properties it is necessary to know the value of T_{mod} given by equation (18) for the strain rate and temperature at AB, but at this stage T_{AB} is not known. It is therefore assumed to start that T_{AB} is equal to the work temperature T_W and this is used together with $\dot{\epsilon}_{AB}$ ($\dot{\gamma}_{AB}/\sqrt{3}$) in equation (18) with ν again taken as 0.09 and $\dot{\epsilon}_0$ as 1/s to give the first estimate of T_{mod} at AB. The corresponding values of σ_1 and n are then found and used in equation (21) to give k_{AB} which is substituted in equation (8) to give the resultant cutting force R with the required value of θ found from equation (5). The required forces are found from equation (8) with the angle difference $(\lambda - \alpha)$ found from equation (7). Equations (9)–(12) can now be used to calculate T_{AB} with S given by equation (20) and K by the appropriate equation similar to equation (22), with T taken as $T_{AB} = T_W$ (°C) and ρ again taken as 7862 kg/m³. The calculations are repeated using this value of T_{AB} as the new starting estimate of the temperature at AB and this process is continued until the difference between the starting estimate of T_{AB} and the calculated value differs by less than 0.1 K. The forces, stresses, etc. at this converged temperature value are taken as the appropriate values for the assumed ϕ and the resolved shear stress at the tool/chip interface τ_{int} is found from

$$\tau_{int} = F/hw, \tag{23}$$

with the tool–chip contact length h given by equation (14). The temperature and strain rate at the tool/chip interface are found from equations (13), (15), (16) and (17). An iterative procedure is again necessary in calculating chip temperatures as the thermal properties are temperature dependent. In the method used, the first estimate of mean chip temperature needed for finding S from equation (20) is taken as equal to T_{AB} and then equation (17) used to calculate T_C. The process is then repeated using this value of T_C added to T_{AB} as the new estimate and continued until the difference between the estimated and calculated values of $(T_{AB} + T_C)$ is less than 0.1 K. Having obtained this value equations (15) and (16) are used to find T_{int} with K in the expression for R_T taken as that corresponding to $(T_{AB}+T_C)$. The value of T_{mod} at the tool/chip interface is found by substituting this value of T_{int} (K) together with the corresponding value of $\dot{\epsilon}_{int}$ $(\dot{\gamma}_{int}/\sqrt{3})$ into equation (18). It is now assumed that the shear flow stress in the chip at the tool/chip interface k_{chip} is given by

$$k_{chip} = \sigma_1/\sqrt{3}, \tag{24}$$

where σ_1 corresponds to the value of T_{mod} at the interface. This equation neglects the influence of strain on flow stress above a strain of 1 (normally $\epsilon_{int} \gg 1$) which is a reasonable approximation, at least for plain carbon steels which tend to strain-harden far less at higher strains. However, in a more complete theory, account should be taken of strain and some preliminary work has already been done on this by Bao & Stevenson (1975) in considering the machining of aluminium.

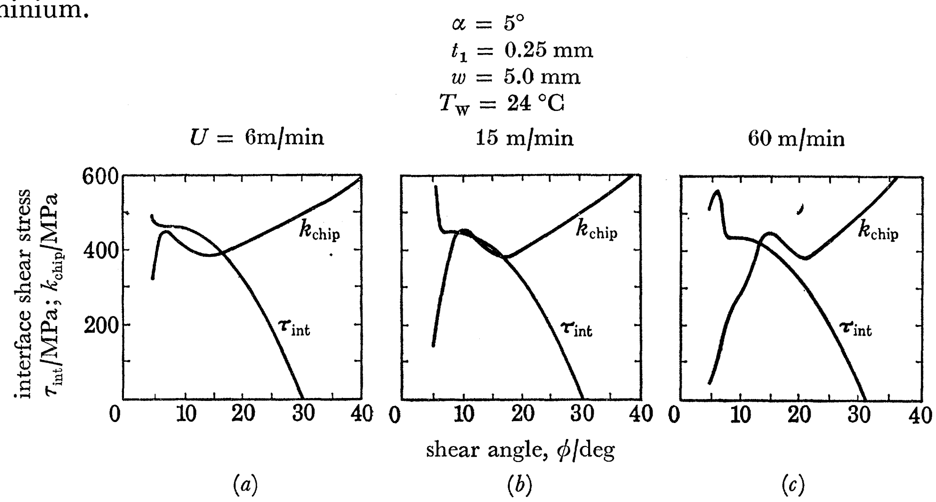

FIGURE 5. Curves of shear stress at tool/chip interface showing how solution point values of shear angle are obtained.

Typical curves of τ_{int} and k_{chip} against ϕ obtained in this way are given in figure 5. The solution for ϕ is taken at the intercept of the two curves. Where there is more than one intercept, as in figure 5*b*, the intercept furthest to the right is taken as this is the first equilibrium solution reached as ϕ decreases from its relatively high value at the start of a cut. It can be seen that depending upon the cutting conditions the intercept can occur in the range of k_{chip} values obtained from the part of the σ_1 curve (figure 3) to the left of A (figure 5*a*) between A and B (blue-brittle range) (figure 5*b*) and to the right of B (figure 5*c*). In this example the change from one type of intercept to the next results from the change in cutting speed, an increase in cutting speed increasing T_{mod} at the tool/chip interface. Changes in rake angle and undeformed chip thickness can act in a similar way by varying T_{mod} at the interface.

A computer program has been written for making the above calculations, including plotting of the τ_{int} and k_{chip} curves and determining their intercepts, that is, the solution points. Predicted and experimental results for a range of cutting conditions and plain carbon steels are given in figures 6–8. Where the intercepts occur in the blue-brittle range (figure 5*b*) the lines representing the predicted results are shown broken. The experimental values of λ were determined from equation (2) using values of F_C and F_T measured with a cutting-force dynamometer, and experimental values of ϕ from the geometric relation (figure 1)

$$\tan\phi = \frac{(t_1/t_2)\cos\alpha}{1-(t_1/t_2)\sin\alpha}, \tag{25}$$

using measured values of chip thickness t_2. The experimental values of T_{int} given in figure 7 were measured by using a tool/work thermocouple. In all cases the predicted results have been calculated using flow stress properties appropriate to the carbon content of the work material used in the machining tests from which the experimental results were taken.† In the machining experiments from which the experimental results have been taken no lubricant was used and the cutting tool material was tungsten carbide.

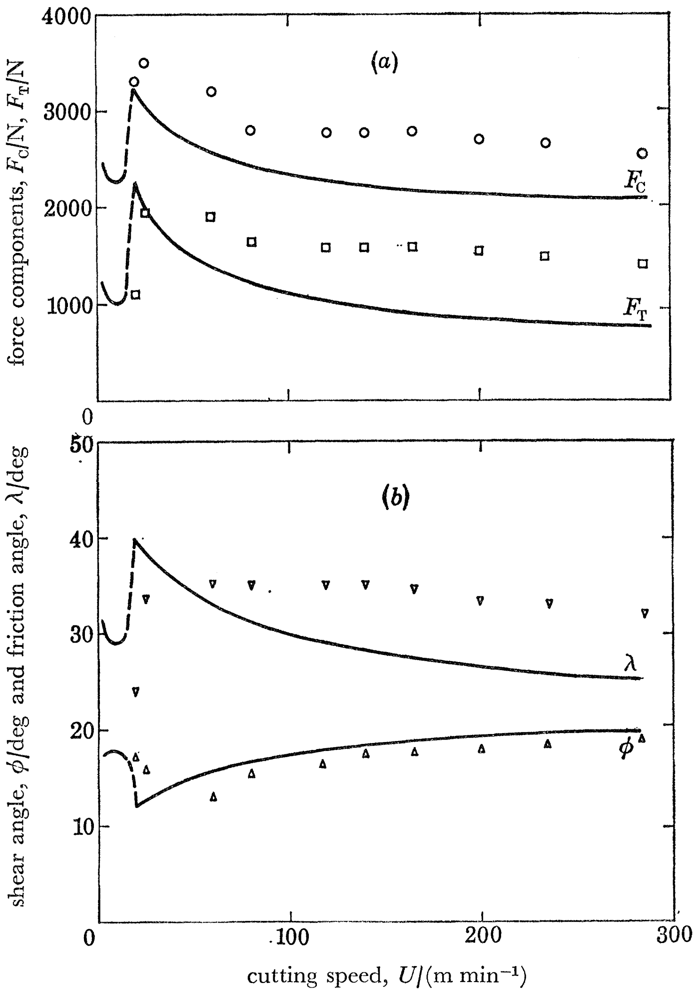

FIGURE 6. Predicted and experimental (Stevenson & Duncan 1973) cutting forces and shear and friction angles for a 0.19% plain carbon steel. $\alpha = 5°$; $t_1 = 0.25$ mm; $w = 5.0$ mm.

The results given in figures 6 and 7 show that the theory predicts the observed trends in cutting forces, interface temperature and shear and friction angles and that the quantitative agreement between the predicted and experimental values of temperature and shear angle is encouragingly good. The cutting forces are underestimated by the theory but this can be partly explained as shown by Tay (1973) by noting that the length of AB, $l = t_1/\sin\phi$ (figure 1), used

† This neglects variations in flow stress properties resulting, for example, from possible differences in grain size and degree of prior plastic working between the materials used in the high speed compression tests of Oyane *et al.* (1967) and the machining tests. Ideally high strain rate flow stress data for the material actually machined should be used in making predictions and the authors are at present taking part in such an investigation.

in making predictions, underestimates the true length of AB by some 10–15 %. It is the relatively large percentage underestimate of F_T compared with F_C which causes λ to be underestimated. It is of considerable interest that the theory predicts cutting force against speed curves which show that the forces first decrease then increase and then decrease again with increase in speed which is the characteristic shape observed experimentally for this kind of material by many workers. This can be explained by considering the changing conditions at the tool/chip interface with increase in cutting speed. For intercepts to the left of A (figure 3) and to the right of B the increase in temperature at the tool/chip interface resulting from an increase in speed, which in spite of the corresponding increase in the strain rate always increases T_{mod}, lowers the flow stress of the chip and hence increases ϕ (decreases λ) and decreases cutting forces.

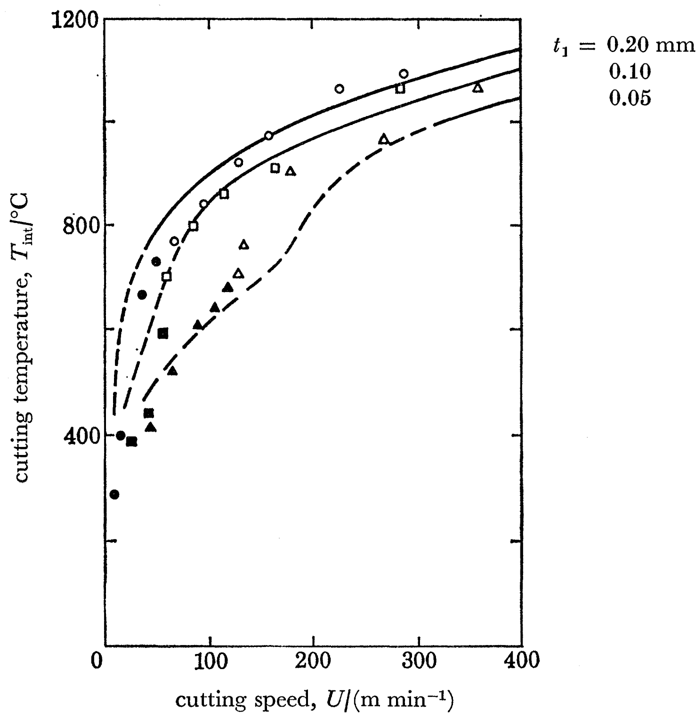

FIGURE 7. Predicted and experimental (OECD–CIRP Seminar on Metal Cutting (1966)) tool/chip interface temperatures for a 0.44% plain carbon steel. △, $t_1 = 0.05$ mm; □, $t_1 = 0.10$ mm; ○, $t_1 = 0.20$ mm; $\alpha = 5°$; $w = 3.0$ mm; $T_W = 20$ °C.

For intercepts within the blue-brittle range an increase in temperature with increase in speed increases the flow stress, reduces ϕ and increases the cutting forces. When the intercepts are in the blue-brittle range it is possible that the flow stress within the chip away from the tool/chip interface, where the temperatures will be lower, will be less than at the interface. It has been suggested by Hastings *et al.* (1974) that the practical consequence of this could be the formation of a built-up edge on the tool cutting face with flow occurring not at the interface as assumed in the model (figure 1) used in the analysis but within the chip. The built-up edge is of course a well known phenomenon in practice and Shaw, Usui & Smith (1962) have previously suggested that its occurrence might be associated with the blue-brittle effect when machining steels. In making the machining tests from which the experimental results in figures 6 and 7 were obtained it was observed that for the results in figure 6 a built-up edge occurred for speeds below

60 m/min while for the tests in figure 7 'filled-in' symbols have been used to signifiy the occurrence of a built-up edge in the tests. These results are close enough to the predicted built-up edge (blue-brittle) range indicated by the broken lines in figures 6 and 7 to give some support to the proposed mechanism.

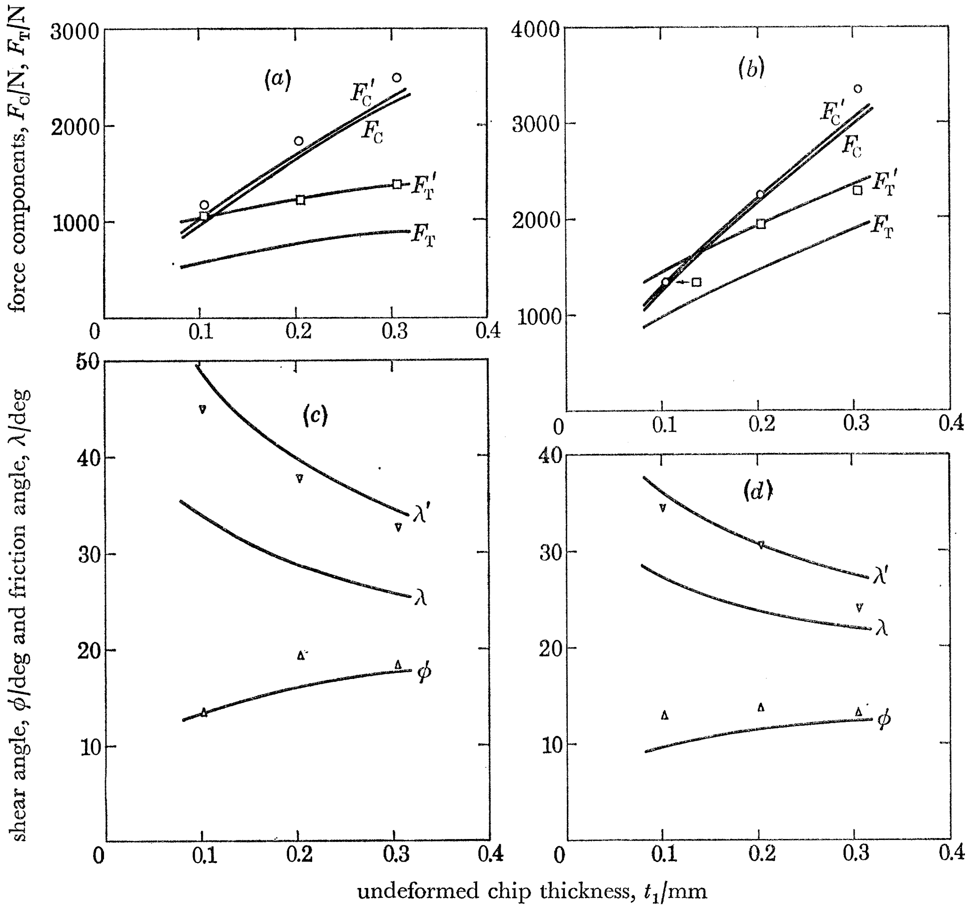

FIGURE 8. Predicted and experimental (Kececioglu (1958)) cutting forces and shear and friction angles for a 0.144% plain carbon steel. (*a*, *c*), $\alpha = 4°$; (*b*, *d*) $\alpha = -10°$. (*a–d*) $U = 194.4$ m/min; $w = 4.29$ mm.

The results in figure 8, which compare the predicted and experimentally observed variations of λ with undeformed chip thickness t_1 and tool rake angle α, were selected for conditions above the blue-brittle range ($T_{mod} > 650$) to ensure that there was no built-up edge to modify the assumed model of chip formation. The theory predicts that an increase in t_1 will decrease λ and increase ϕ while an increase in α will increase both λ and ϕ and the experimental results support these trends. The quantitative agreement for ϕ is again good but λ is underestimated in the same way as for the results in figure 6 the reason again being the relatively large underestimate of F_T compared with F_C. In trying to explain these discrepancies it is worthwhile to consider possible differences between the assumptions made in deriving the machining theory and the actual conditions in the machining experiments. It is clear that no tool can be perfectly sharp as assumed in the analysis but measurements made on the specially prepared tools normally used in machining experiments show that the radius on the tool cutting edge is less than 0.01 mm. This is ten times smaller than the smallest value of t_1 used in the experiments considered and could not explain the differences between the predicted and experimental results. In order to maintain tools as near to perfectly sharp as possible, machining experiments for measuring cutting forces, etc., are usually of short duration (30 s or less). However, it is now well established (see, for example, Taylor 1963; Trigger 1963) that there is a very high initial rate of wear at the tool clearance face which results in a flat surface on this face (approximately parallel to the cutting velocity) which reaches a length, measured in the cutting velocity direction, of between 0.04 and 0.06 mm in less than 20 s of machining. For practical cutting speeds (i.e. above the built-up edge range) the size of this initial flank wear is found to be largely independent of cutting conditions. Cutting tools are normally mounted in such a way that elastic deflexions resulting from the cutting forces tend to press the tool clearance face against the newly machined surface and it is interesting to estimate the force which would result if the flank wear was say 0.05 mm. The problem approximates to the plane strain indentation of a semi-infinite block by a flat ended punch (Hill, Lee & Tupper 1947) and if it is assumed that the pressure is sufficient for the work material in contact with the tool to be at the point of yielding then its value is approximately $5.2k$, where k is the appropriate shear flow stress. It is difficult to estimate T_{mod} in this region but from figure 3 it can be seen that σ_1 can be taken as 700 MPa without introducing large errors for a wide range of T_{mod} values which might be expected to include the appropriate values. Assuming now that $k = 700/\sqrt{3}$ MPa and noting that the width of the contacting surfaces w is 4.29 mm the upwards force on the base of the tool is found to be $\Delta F_T \approx$ 450 N. Although the above calculation neglects the friction between the flank and the newly machined surface some estimate can be made of the friction force $\Delta F_C = \mu \Delta F_T$ by choosing a suitable value of μ. Clearly μ cannot be greater than 1/5.2 and a value of 0.1 has been taken, i.e. $F_C = 45$ N. These values have been added to F_C and F_T in figure 8 to give F_C' and F_T' and the corresponding values of λ' determined. The agreement between the modified predicted results and the experimental results can be seen to be excellent. The predicted results in figure 6 could be similarly improved.

5. Minimum work criterion

In the machining calculations described above δ, the ratio of the average thickness of the tool/chip interface plastic zone to the chip thickness, was assumed constant and equal to 0.05, this being the value found experimentally by Stevenson & Duncan (1973) from machining tests in which only the cutting speed was varied. Similar results showing the influence of undeformed chip thickness and other parameters on δ do not appear to be available. In what follows a method of predicting δ from the machining analysis is described and a comparison is made with results obtained from experiments in which both cutting speed and undeformed chip thickness were varied.

From the machining theory it can be seen that as δ is reduced the calculated values of strain rate and temperature at the tool/chip interface both increase, with $\dot{\gamma}_{int}$ tending to infinity and T_{int} to some finite value as δ approaches zero. The combined effect of these strain rate and temperature changes with δ is to give curves of interface T_{mod} of the shape shown in figure 9 with T_{mod} passing through a maximum at some particular value of δ. The results show that the maximum value of T_{mod} increases with increase in cutting speed U and undeformed chip thickness t_1 while the corresponding values of δ decrease. If attention is limited to conditions where T_{mod} at the interface is above the blue-brittle range then the curves for the shear flow stress k_{chip}, the chip thickness t_2 and the rate of work, both total $F_C U$ and frictional FV, corresponding to a typical T_{mod} curve are as shown in figure 10. The minimum value of k_{chip} must of course coincide with the maximum in T_{mod} and as far as can be judged within the accuracy of the calculations this is also the case for the minimum values of t_2 and $F_C U$ and FV. The possibility that in practice δ will take up values satisfying this minimum work condition is now considered.

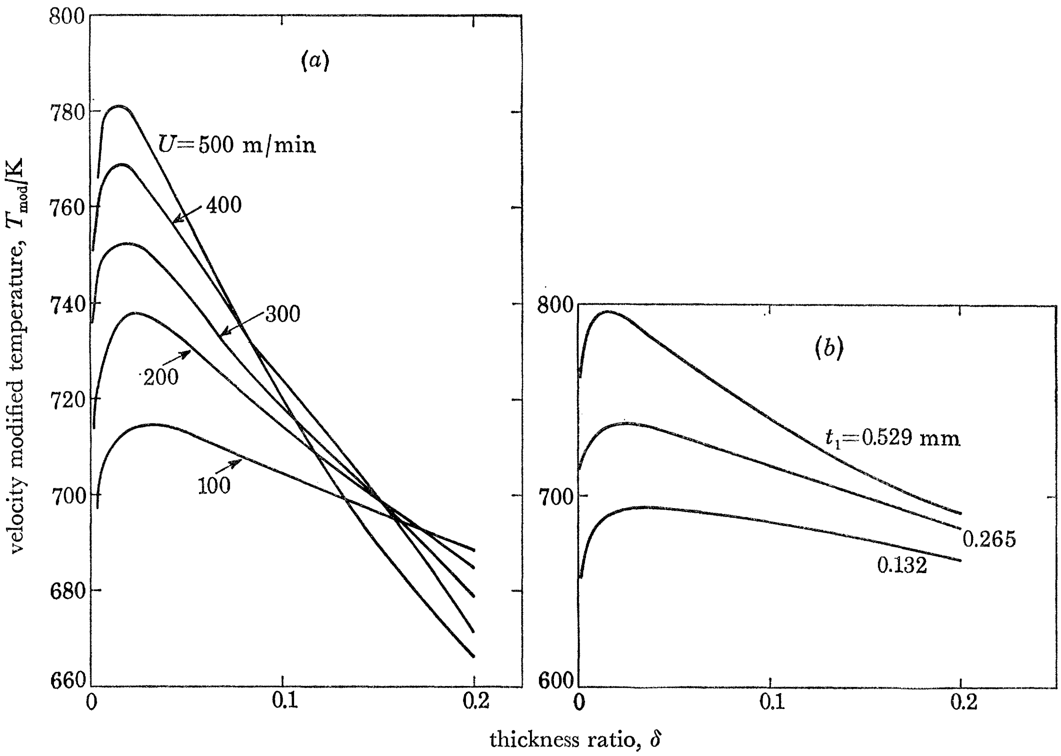

FIGURE 9. Curves of velocity modified temperature. (*a*) $\alpha = 5°$; $t_1 = 0.265$ mm; $w = 3.0$ mm. (*b*) $\alpha = 5°$; $U = 200$ m/min; $w = 3.0$ mm.

Values of t_2 and δt_2 predicted from the machining theory by assuming minimum work are represented by the full lines in figures 11 and 12 which also give experimental results. The experiments were made on a nominal 0.15–0.25% plain carbon steel and for the actual test material a chemical analysis gave 0.25% C, 0.60% Mn, 0.30% P plus negligible amounts of trace elements, and this composition was used in making the predictions. In the experiments a conventional turning process was used with the main cutting edge of the tool set normal to the cutting and feed (parallel to the machined bar axis) velocities. The rake angle of the cutting tool (tungsten carbide) was 5° with the width of cut 3.0 mm and the ranges of undeformed chip thickness (0.132, 0.265, 0.529 mm) and cutting speed (100–450 m/min) covered were determined by power limitations at high speeds and by built-up edge and discontinuous chip formation at speeds below 100 m/min. The conditions were only approximately orthogonal as a bar and not a tube was used and therefore there was some cutting at the small secondary cutting edge approximately parallel to the feed direction. Also for the largest t_1 value (0.529 mm) the ratio of w/t_1 was only just over 5, which is less than the value usually suggested for approximate plane strain conditions. However, measurements of the width of the chip showed that it was only slightly greater than 3.0 mm. The conditions in the experiments therefore approximated closely to those assumed in the machining analysis. The experimental values of t_2 and δt_2 were measured on a Nikon Shadowgraph, equipped with a measuring stage, from chip sections which had been mounted, polished and etched using standard metallurgical techniques. For each test condition two chip specimens were obtained to show possible variations of t_2 and δt_2 during the tests and two measurements were made on each specimen thus giving four readings in all. The method of measurement can be seen from the photomicrograph of a typical chip section given in figure 13. The chip thickness t_2 was taken as the distance from the back face of the chip to a line drawn to average out the relatively rough outer chip surface. From the photomicrograph (figure 13) it can be seen that in the bulk of the chip the grains exhibit a markedly

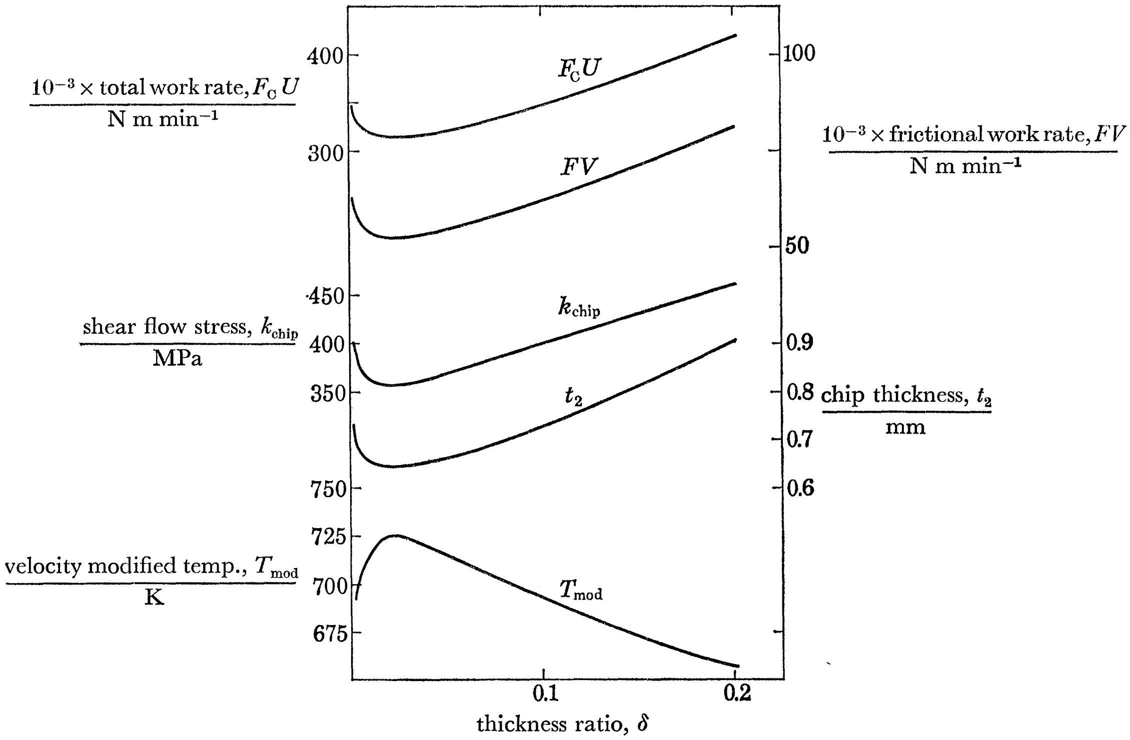

FIGURE 10. Curves of shear flow stress, chip thickness and rate of total and frictional work corresponding to a typical velocity modified temperature curve. $\alpha = 5°$; $U = 250$ m/min; $t_1 = 0.265$ mm; $w = 3.0$ mm.

preferred orientation which results from the deformation in the main plastic zone in which the chip is formed. Material which has also passed through the plastic zone adjacent to the tool/chip interface can be identified reasonably clearly by noting where the grains have been swept back relative to the rest of the microstructure as indicated by the line in figure 13. The distance from the back face of the chip to this line will give the maximum thickness of the tool/chip interface plastic zone and from the experimental results of Zorev (1966) and Tay *et al.* (1974), who used 'quick-stop' chip sections to measure the size and shape of this zone, it appears that the average thickness δt_2 is approximately half the maximum thickness. The experimental results for δt_2 given in figure 12 have therefore been taken as half the measured thicknesses. For most conditions the four readings of t_2 and δt_2 were fairly close and only the largest and smallest values have been given in figures 11 and 12.

The agreement between the values of t_2 predicted by assuming minimum work (figure 11) and the experimental values is good with the predictions showing, in agreement with experiment, that t_2 decreases with increase in cutting speed and increases rather less than in direct proportion to t_1. The predicted values of δt_2 show a decrease in δt_2 with increase in cutting speed and decrease in t_1 although the latter is far less than would be given by a constant δ. The experimental values of δt_2 show reasonable agreement with the predictions but indicate a somewhat larger change with t_1 than predicted; they are too scattered to give a clear confirmation of the predicted decrease of δt_2 with speed. It should be noted that for the smallest t_1 value (0.132 mm) there

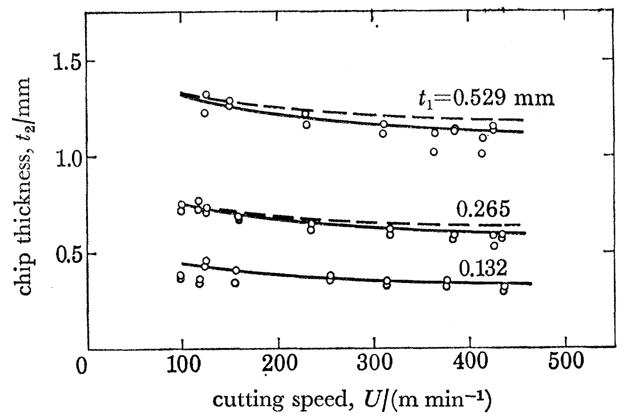

FIGURE 11. Predicted and experimental chip thicknesses: full lines give minimum work values and broken lines $\delta = 0.05$ values. $\alpha = 5°$; $w = 3.0$ mm

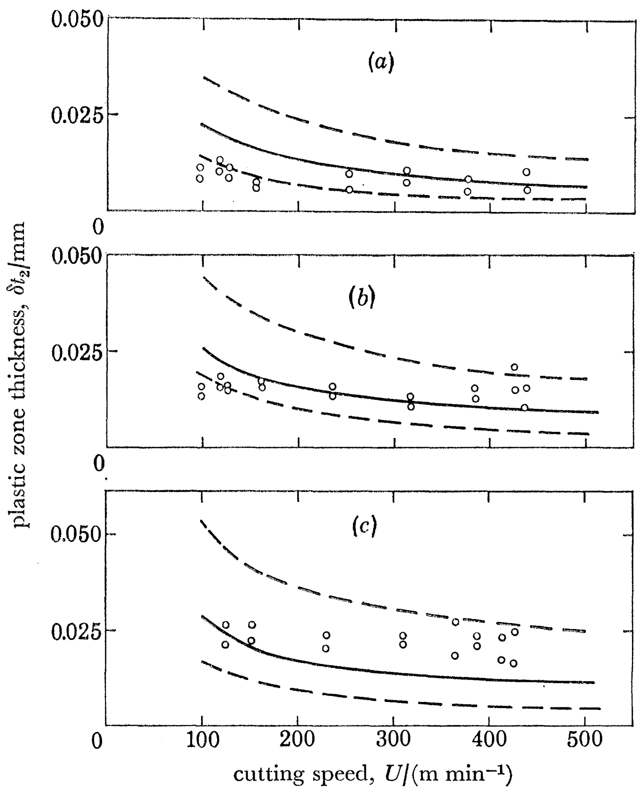

FIGURE 12. Predicted and experimental tool/chip interface plastic zone thicknesses: full lines give minimum work values and broken lines 1% deviation from minimum work values. (a) $t_1 = 0.132$ mm; (b) $t_1 = 0.265$ mm; (c) $t_1 = 0.529$ mm. (a–c) $\alpha = 5°$, $w = 3.0$ mm.

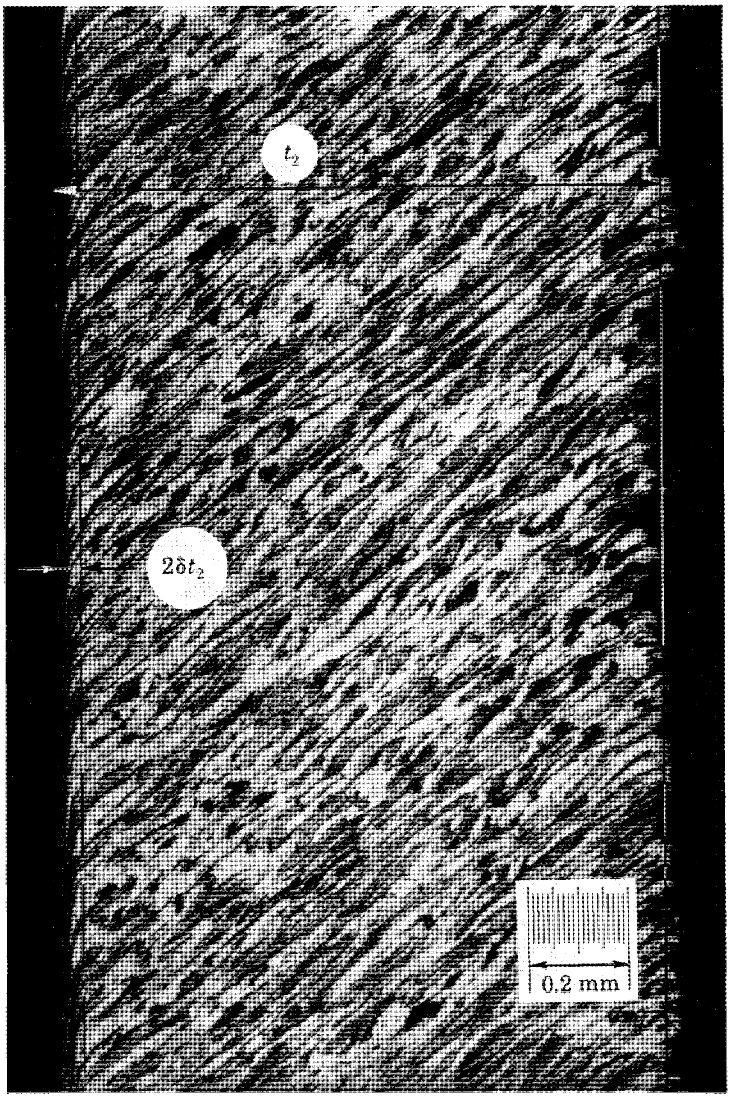

FIGURE 13. Photomicrograph of chip section.

were traces of built-up edge on the chip sections for speeds below 150 m/min and this could have affected the experimental results in this range. The curves in figure 10 are very flat in the regions of the turning points which means that the parameters on the vertical axis such as t_2 are far less sensitive than is δ to small deviations from the minimum work condition. The broken lines in figure 12 give δt_2 values corresponding to a 1 per cent deviation from minimum work on the total rate of work $(F_C U)$ curves and it can be seen that most of the experimental values of δt_2 are encompassed within these bounds. If a 2 % deviation had been used then all of the experimental points would have been encompassed. The corresponding deviations in t_2 are too small to be shown in figure 11. In considering previous applications of the machining theory (see, for example, Hastings *et al.* 1974) including the calculations described in the previous section of this paper in which δ was assumed constant and equal to 0.05 it can be seen that the relative insensitivity of parameters such as t_2 to δ in the minimum work regions is fortunate. The broken lines in figure 11 give the predicted values of t_2 corresponding to a δ of 0.05 and it can be seen

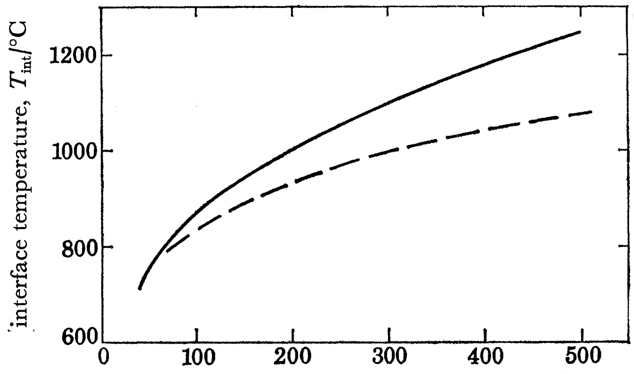

FIGURE 14. Predicted tool/chip interface temperatures: full line gives minimum work values and broken line $\delta = 0.05$ values. $\alpha = 5°$; $t_1 = 0.265$ mm; $w = 3.0$ mm.

that these are not very different from the minimum work values; for the smallest t_1 the differences are too small to be shown. This insensitivity to δ can be contrasted to the marked influence on t_2, cutting forces, etc., of variations in the strain rate constant C in equation (3) and in the material flow stress properties, t_2 being particularly sensitive to variations in the strain-hardening index n. From the calculations so far made for plain carbon steels it appears that for cutting conditions within the normal range it is sufficiently accurate for most purposes to use a constant value of δ, thus reducing the amount of computation, although 0.02 might be a more appropriate value than 0.05. An exception to this has been found in calculating interface temperatures which show a greater dependence on δ as can be seen from figure 14 which gives minimum work values of T_{int} and also values corresponding to a δ of 0.05. When machining with tungsten carbide tools in the normal cutting speed range, tool wear occurs mainly from a diffusion mechanism which is highly dependent on temperature, and differences of the order shown in figure 14 would be extremely important in attempting to predict the effective life of the cutting tool.

It can be concluded that for the rather limited range of conditions considered the results presented are reasonably consistent with the proposition that the frictional conditions at the tool/chip interface are determined by a minimum work criterion. Further experiments are now being planned to see if this still applies when a much wider range of cutting conditions and work materials are considered. Also attempts are being made to improve the approximate machining theory by taking account of the actual sliding which can occur at the tool/chip interface and by including the effects of the tool shape (heat sink) and thermal properties of the tool material in the temperature calculations.

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References

Bao, H. & Stevenson, M. G. 1975 Report, School of Mechanical and Industrial Engineering, University of New South Wales.
Boothroyd, G. 1963 *Proc. Instn. Mech. Engrs.* **177**, 789.
Campbell, J. D. & Ferguson, W. G. 1970 *Phil. Mag.* **21**, 63.
Christopherson, D. G., Oxley, P. L. B. and Palmer, W. B. 1958 *Engineering* **186**, 113.
Fenton, R. G. & Oxley, P. L. B. 1969 *Proc. Instn mech. Engrs* **183**, 417.
Fenton, R. G. & Oxley, P. L. B. 1970 *Proc. Instn mech. Engrs* **184**, 927.
Hastings, W. F., Oxley, P. L. B. & Stevenson, M. G. 1974 *Proc. Instn mech. Engrs* **188**, 245.
Hill, R., Lee, E. H. & Tupper, S. J. 1947 *Proc. R. Soc. Lond.* A **188**, 273.
Kececioglu, D. 1958 *Trans. A.S.M.E.* **80**, 149.
Lee, E. H. & Schaffer, B. W. 1951 *J. appl. Mech., Trans. A.S.M.E.* **73**, 405.
MacGregor, C. W. & Fisher, J. C. 1946 *J. appl. Mech., Trans. A.S.M.E.* **13**, A11.
Merchant, M. E. 1945 *J. appl. Phys.* **16**, 267.
Oyane, M., Takashima, F., Osakada, K. & Tanaka, H. 1967 *10th Japanese Congress on Testing Materials*, p. 72.
Palmer, W. B. & Oxley, P. L. B. 1959 *Proc. Instn mech. Engrs* **173**, 623.
Proceedings OECD–CIRP Seminar on Metal Cutting, Sept. 1966.
Proceedings of Conference on Mechanical Properties at High Rates of Strain, Oxford 1974, (Inst of Phys.).
Roth, R. N. & Oxley, P. L. B. 1972 *J. Mech. Engng Sci.* **14**, 85.
Shaw, M. C., Usui, E. & Smith, P. A. 1962 *J. Engng. Ind. Trans. A.S.M.E.* **84**, 89.
Stevenson, M. G. & Duncan, K. R. 1973 *J. Iron Steel Inst.*, **211**, 710.
Stevenson, M. G. & Oxley, P. L. B. 1970 *Proc. Instn mech. Engrs* **184**, 561.
Tay, A. O. 1973 Ph.D. Thesis, University of New South Wales.
Tay, A. O., Stevenson, M. G. & de Vahl Davis, G. 1974 *Proc. Instn mech. Engrs* **188**, 627.
Taylor, J. 1963 Proceedings International Production Engineering Research Conference, Pittsburgh, (A.S.M.E.), p. 130.
Trigger, K. J. 1963 Proceedings International Production Engineering Research Conference, Pittsburgh, (A.S.M.E.), p. 95.
Wallace, P. W. & Boothroyd, G. 1964 *J. mech. Engng Sci.* **6**, 74.
Weiner, J. H. 1955 *Trans. A.S.M.E.* **77**, 133.
Woolman, J. & Mottram, R. A. 1964 *The mechanical and physical properties of the British En steels*, British Iron and Steel Research Association: Pergamon Press.
Zorev, N. N. 1963 Proceedings International Production Engineering Research Conference, Pittsburgh, (A.S.M.E.) p. 42.
Zorev, N. N. 1966 *Metal cutting mechanics*. Oxford: Pergamon Press (English translation).

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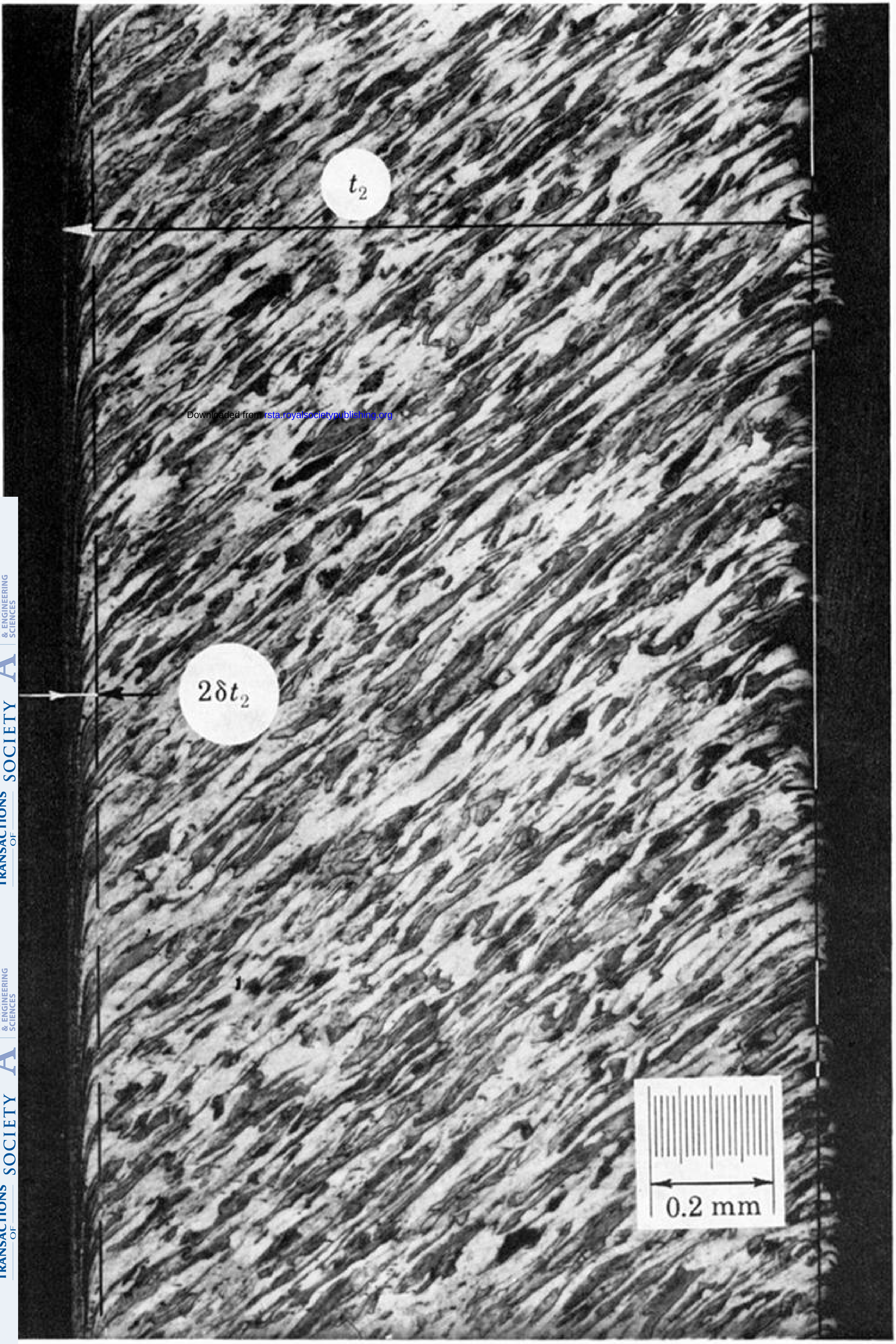

FIGURE 13. Photomicrograph of chip section.